Leading the Lorenz system toward the prescribed regime by model predictive control combined with data assimilation

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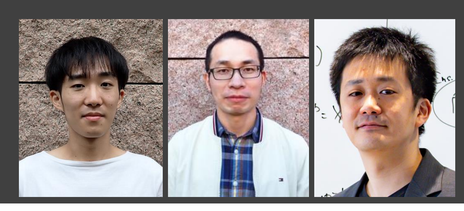


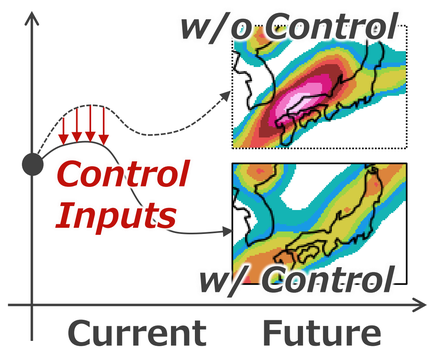
Overview

- This study proposes a Control Simulation Experiment (CSE) using model predictive control (MPC).
- Our experiments with Lorenz-63 have shown that the proposed method successfully leads the system toward the prescribed regime for constraints.

Introduction

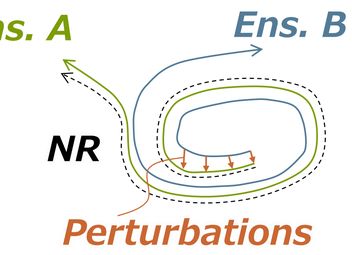
Weather-related disasters

- If we could lead the weather toward non-disastrous regime, the caused damage could be reduced.
- Various **constraints** need to be considered. (e.g., **feasible control input magnitudes**, **prediction length**)



Control Simulation Experiment (CSE)

- Miyoshi and Sun (2022) have proposed the CSE, which tries to lead a system toward preferable regimes by adding small perturbations into the nature run (NR).
- Explicit constraints is more difficult.

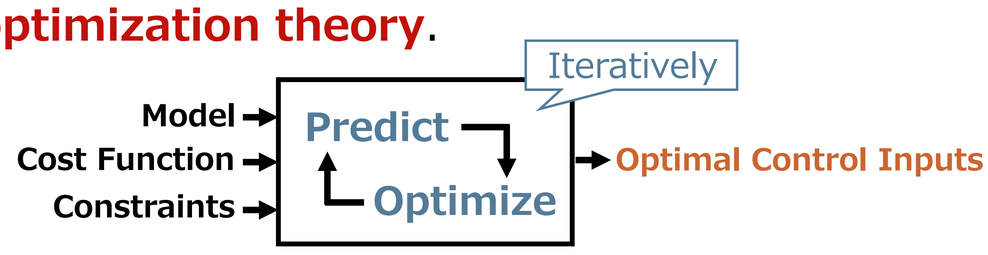


➡ **Applying MPC which enables flexible control considering the constraints to CSE.**

Model Predictive Control combined with EnKF

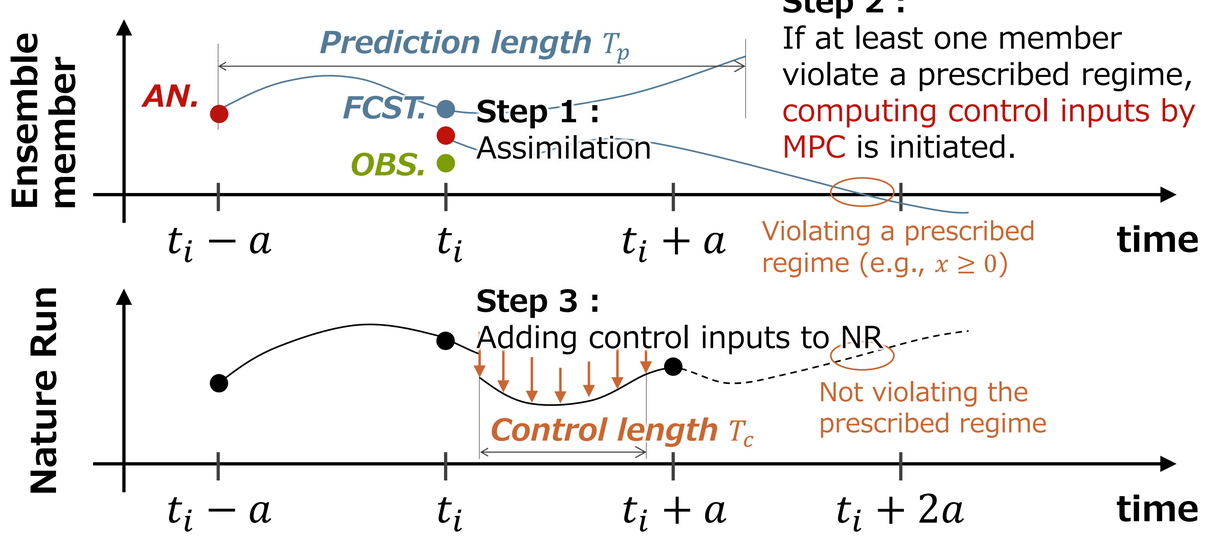
What's MPC ?

- MPC is a method for obtaining optimal control inputs based on **model-based prediction** and **optimization theory**.



- Variational problem **with constraints**.
- Solved by method of Lagrange multiplier.

CSE with MPC



Experiments & Results

- **Leading the positive regime of Lorenz-63**
- ETKF (3 members)
- $dt = 0.01$
- Pred. length : $0.2 = 20$ steps
- Cntl. length : $0.07 = 7$ steps

Model

$$\dot{x} = -s(x - y) + u_x$$
$$\dot{y} = -xz + rx - y + u_y$$
$$\dot{z} = xy - bz + u_z$$

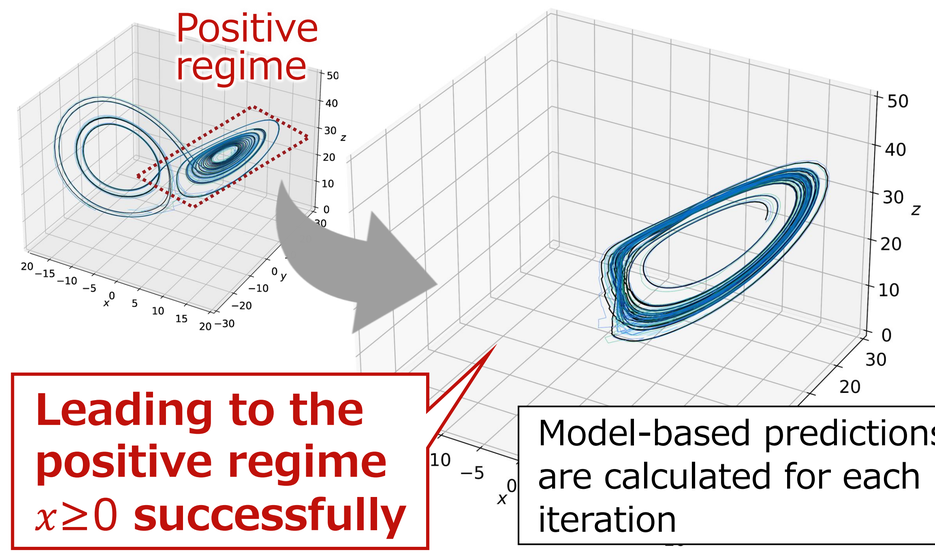
Cost Function

$$J = \int_t^{t+T} \frac{1}{2}\mathbf{u}^T\mathbf{u}\, d\tau$$

Constraints

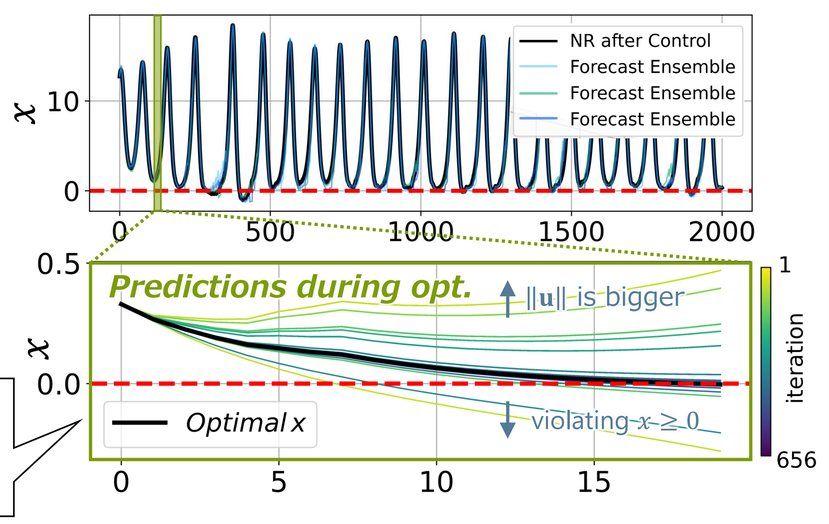
$$x \geq 0, \qquad \|\mathbf{u}\| \leq 20$$

State variables

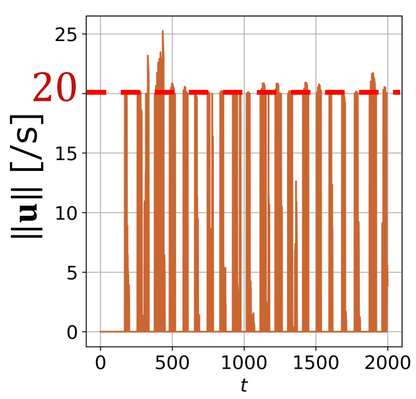


Leading to the positive regime $x \geq 0$ successfully

Model-based predictions are calculated for each iteration



Control Inputs



$\|\boldsymbol{u}\| \leq 20$ can be almost satisfied

Optimization algorithm (e.g., Newton's method) finds control inputs that minimizes the cost function